

Double Beta Decay in pn-QRPA Model with Isospin and SU(4) Symmetry Constraints

F. Krmpotić[†]

Departamento de Física, Facultad de Ciencias Exactas,

Universidad Nacional de La Plata, C. C. 67, 1900 La Plata, Argentina

S. Shelly Sharma^{*}

Departamento de Física, Universidade Estadual de Londrina, Londrina, Parana, and

Instituto de Física Teórica, Rua Pamplona-145 CEP 01405 - São Paulo, Brazil

Abstract

The transition matrix elements for the $0^+ \rightarrow 0^+$ double beta decays are calculated for ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te nuclei, using a δ -interaction. As a guide, to fix the particle-particle interaction strengths, we exploit the fact that the missing symmetries of the mean field approximation are restored in the random phase approximation by the residual interaction. Thus, the $T = 1$, $S = 0$ and $T = 0$, $S = 1$ coupling strengths have been estimated by invoking the partial restoration of the isospin and Wigner SU(4) symmetries, respectively. When this recipe is strictly applied, the calculation is consistent with the experimental limit for the 2ν lifetime of ^{48}Ca and it also correctly reproduces the 2ν lifetime of ^{82}Se . In this way, however, the two-neutrino matrix elements for the remaining nuclei are either underestimated (for ^{76}Ge and ^{100}Mo) or overestimated (for ^{128}Te and ^{130}Te) approximately by a factor of 3. With a comparatively small variation ($< 10\%$) of the spin-triplet parameter, near the value suggested by the SU(4) symmetry, it is possible to reproduce the measured $T_{1/2}^{2\nu}$ in all the cases. The upper limit for the effective neutrino mass, as obtained from the theoretical estimates of 0ν matrix elements, is $< m_\nu > \cong 1$ eV. The dependence of the nuclear matrix elements on the size of the configuration space has been also analyzed.

[†]Fellow of the CONICET from Argentina

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1 Introduction

The odd-odd isobar, within the isobaric triplet (N, Z) , $(N - 1, Z + 1)$, $(N - 2, Z + 2)$, generally has a higher mass than its even-even neighbors due to the pairing energy. As such, two consecutive β decays are energetically forbidden. Yet, the initial nucleus (N, Z) may decay to the final nucleus $(N - 2, Z + 2)$ through virtual excitations of the intermediate nucleus $(N - 1, Z + 1)$, causing a double beta ($\beta\beta$) decay process. In case the lepton number is strictly conserved the neutrino is a Dirac fermion ($\nu \neq \bar{\nu}$) and the two-neutrino mode ($\beta\beta_{2\nu}$) is the only possible mode of disintegration. On the other hand, if this conservation is violated, the neutrino is a Majorana particle ($\nu = \bar{\nu}$) and the neutrinoless double beta decay ($\beta\beta_{0\nu}$) also can occur.

Both the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ -decay processes have attracted much attention during the last decade. This is mainly because the neutrinoless decay mode constitutes the most critical touchstone for various gauge models that go beyond the standard $SU(2)_L \times U(1)$ gauge model of the electroweak interaction. The $\beta\beta_{0\nu}$ decay rate depends on several unknown parameters (neutrino mass, majoron coupling, the coupling constants of the right-handed components of the weak Hamiltonian, etc.) and the only way to put these in evidence is by having sufficient command over the nuclear structure. It is precisely at this point that the $\beta\beta_{2\nu}$ decay mode is important. A comparison between experiment and theory for the $\beta\beta_{2\nu}$ -decay, provides a measure of confidence one may have in the nuclear wave functions employed for extracting the unknown parameters from $\beta\beta_{0\nu}$ lifetime measurements.

In recent years, following the work of Vogel and Zirnbauer [1], the proton-neutron (pn) quasiparticle random phase approximation (QRPA) has turned out to be the most popular model for calculating the nuclear wave functions involved in the $\beta\beta$ -transitions [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. The QRPA calculations have shown that the ground state correlations (GSC), induced by the residual pn interaction in the particle-particle (PP) channel, play an essential role in quenching the $\beta\beta_{2\nu}$ decay probabilities. Regarding the sensitivity of $\beta\beta_{0\nu}$ nuclear moments, $\mathcal{M}_{0\nu}$, to the PP interaction, no consensus has been reached so far. The results obtained by the Pasadena group [4], with a residual δ interaction, seem to show that $\mathcal{M}_{0\nu}$ are rather sensitive to this force and the

^{76}Ge lifetime measurement [23] ($T_{1/2}^{0\nu} > 1.4 \times 10^{24} \text{yr}$) yields the upper bound on the effective neutrino mass, $\langle m_\nu \rangle$, varying between 4.4 and 10 eV (see table 1). On the other hand, the calculations done by Tübingen and Heidelberg groups with a realistic interaction (the G-matrix of the Paris and Bonn potentials) [3, 8, 18, 20], suggest that the $\mathcal{M}_{0\nu}$ moments are to some extent insensitive to the PP channel, which results in a smaller neutrino mass with more stringent upper limit (between 2.3 eV and 3.1 eV) from the same datum [23]. The difference between the three sets of calculations lies in the importance attached to the role of destructive interference between the allowed ($L = 0$) and the forbidden ($L > 0$) virtual transitions. There is a general agreement that, for the physically acceptable values of PP coupling strength, $L = 0$ contributions are relatively small and those from $L > 0$ processes are significant. But, whereas the Pasadena group believes that the allowed matrix elements are strongly canceled by the forbidden ones, the other two groups claim the cancellation effect to be of minor importance. The exceptions are the results for the $\mathcal{M}_{0\nu}$ moments in ^{48}Ca , ^{100}Mo and ^{128}Te : for the first two nuclei very small nuclear matrix elements have been obtained by Heidelberg group, suggesting that the $L = 0$ matrix elements are sizable, while for the last nucleus all three groups obtain similar results.

An important question in the QRPA calculations is, how to fix the PP interaction strengths within the $S = 0, T = 1$ and $S = 1, T = 0$ channels? Several attempts have been made to calibrate the last one using the experimental data for individual Gamow-Teller (GT) positron decays [4, 6, 7]. The weak point of this procedure is that the distribution of the β^+ strength among low-lying states in odd-odd nuclei is certainly affected by the charge-conserving vibrations, which are not included in the QRPA.¹ In the present work we resort to the restoration of the isospin and spin-isospin SU(4) symmetries to gauge, respectively, the $S = 0, T = 1$ and $S = 1, T = 0$ strengths in the PP channel [9]. Unlike the method mentioned above, this procedure involves the total Fermi (F) and GT strengths, which dependent of the charge-conserving vibrations only very weakly. We are aware, how-

¹The single beta transitions $^{100}\text{Tc} \rightarrow ^{100}\text{Mo}$ and $^{100}\text{Tc} \rightarrow ^{100}\text{Ru}$ have been discussed recently in the standard QRPA [29]. However, from the analysis of the structure of the triplet of low-lying states in ^{101}Mo (cf. ref. [30]), it can be inferred that the collective degrees of freedom should play a very important role in these decays.

ever, that the $SU(4)$ symmetry is badly broken in heavier nuclei like those considered here. As such, before proceeding further, it is necessary to specify what we mean by reconstruction of isospin and $SU(4)$ symmetries in the context of our calculation.

For a system with $N \neq Z$, the isospin and spin-isospin symmetries are violated in the mean field approximation, even if the nuclear hamiltonian commutes with the corresponding excitation operators β^\pm (t_\mp and σt_\mp). But, Thouless [31] and Brown [32] have shown that when a non-dynamical violation occurs in the Hartree-Fock (HF) solution, the RPA induced GSC can be invoked to restore the symmetry [33, 34, 35]. There are subtleties involved in the restoration mechanism: the GSC are not put in evidence explicitly, but only implicitly via their effects on the one-body moments β^\pm between the ground state and the excited states. Besides, for the F excitations and when the isospin non-conserving forces are absent, a self-consistent inclusion of the GSC leads to the following:

- i) all the β^- strength is concentrated in the isobaric analog state (IAS), and
- ii) the β^+ spectrum, which in RPA can be viewed as an extension of the β^- spectrum to negative energies, is totally quenched.

The extent to which the above conditions are fulfilled may be taken as a measure of the isospin symmetry restoration.² Lee [34] has shown that even when the isospin symmetry is dynamically broken by the Coulomb force, the RPA induced GSC result in a near restoration of the isospin symmetry. Besides being spontaneously broken by the HF approximation, the $SU(4)$ symmetry is also dynamically torn down by the spin-orbit field and the supermultiplet-destroying residual interactions. But, as argued below, the last two effects have a tendency to cancel each other.

In the mean field approximation (and due to the spin-orbit splitting) the σt_- operator has roughly equal strengths for the spin flip as compared to non-spin flip transitions. Charge exchange reactions [36, 37, 38] have revealed, still, that the spin-isospin residual forces can to some extent overcome this $SU(4)$ symmetry breaking by transferring a substantial amount of the non-spin flip strength to higher energies (and build up in this way the GT resonance).

²We may point out that it is not possible to obtain similar results in the Tamm-Dancoff approximation (TDA) where the GSC are neglected. As a matter of fact within the TDA the β^- strength is always fragmented and the perturbed β^+ strength remains equal to its unperturbed value.

In fact, the observed mean energy differences between the GT and F resonances can be accounted for by the relation [39]

$$E_{GT} - E_F = \left(26A^{-1/3} - 18.5 \frac{N - Z}{A} \right) MeV,$$

and the displacement of the GT resonance towards the IAS with increasing $N - Z$ may be interpreted as the effect of the residual interaction [39, 40, 41]. As a matter of fact, the first term has the mass dependence of the mean spin-orbit splitting, Δ_{ls} , and within a schematic TDA for the δ interaction (that we use in the present work) one gets [39]

$$E_{GT} - E_F = \left[\Delta_{ls} - (v_t - v_s) \frac{N - Z}{2A} \right] MeV,$$

where v_s and v_t are, respectively, the singlet and the triplet coupling constants. It should be noticed that the GSC are likely to alter this result very little. But, within the RPA the $\sigma\tau_+$ transition strength is strongly quenched and the GT resonance is somewhat narrowed, as compared with the TDA results [12]. As such the global effect of the pn residual interaction on the GT strength is qualitatively similar to the corresponding effect on the F strength, and we say that the SU(4) symmetry is partially restored. Finally a word of caution, the closeness of GT resonance to IAS does not guarantee a complete reconstruction of the SU(4) symmetry.³ For example, the observed GT strength in ^{208}Pb is in a single continuous peak located at the energy of the IAS, but simultaneously the GT resonance is rather broad ($\approx 4MeV$; see ref. [37]).

In the present work we perform a detailed analysis of the $\beta\beta_{0\nu}$ decay rates for several nuclei, always focussing our attention on the $< m_\nu >$ term (mass mechanism). The term containing the right-handed leptonic currents (RHC mechanism) is not considered. Besides that, no reference is made to the neutrinoless $\beta\beta$ decays with single [52, 53], $\beta\beta_{0\nu,B}$, and

³The problem of reconstruction of the Wigner SU(4) symmetry and its relationship with the proton-neutron interaction, has been discussed earlier in the context of the GT transitions by Fujita and Ikeda [42] and by Bohr and Mottelson [43]. Recently Bernabeu et al. [44] have shown that treating the SU(4) symmetry-breaking part of the interaction in perturbation theory leads to $\mathcal{M}_{2\nu}$ moments of correct order of magnitude. Several other manifestations of the SU(4) symmetry in nuclei have also been reported [45, 46, 47, 48, 49, 50, 51].

double [54], $\beta\beta_{0\nu,2B}$, majoron emissions. The $\beta\beta_{0\nu,B}$ decay has been ruled out by the measurement of the Z width [55], as shown by Gonzalez-Garcia and Nir [56], whereas no data is available for the $\beta\beta_{0\nu,2B}$. We may note that the effective couplings between neutrinos and the majorons are easily estimated once the nuclear matrix elements are known. Presently our main aim is to set up an upper limit on the neutrino mass from the 0ν -decay analysis. Since a good understanding of the 0ν -decay requires that we understand the 2ν -decay really well, the $\beta\beta_{2\nu}$ decay rates have been reviewed as well.

The paper is organized as follows. In section 2 we sketch the main formalism and list various formulae needed to compute the half lives for the $0^+ \rightarrow 0^+$ transitions. Section 3 deals with the discussion of some important features of nuclear matrix elements, the comparison of numerical results with the experimental data, and the present limits on the neutrino mass. Finally, the conclusions are drawn in section 4. The evaluation of radial integrals, involved in the 0ν -decay, is shortly reviewed in appendix A.

2 Formalism

To apply the Horie-Sasaki method [57], we write down the Fourier-Bessel expansion of 0ν transition amplitude as

$$\mathcal{M}_{0\nu} = \frac{R}{4\pi} \sum_{S\alpha J^\pi M} \hat{S} \int d\mathbf{q} \mathbf{v}(q; \omega_{\alpha J^\pi}) \langle 0_f^+ | e^{i\mathbf{q}\mathbf{r}_1} \sigma_1^S t_+ | \alpha J^\pi M \rangle \otimes \langle \alpha J^\pi M | e^{-i\mathbf{q}\mathbf{r}_2} \sigma_2^S t_+ | 0_i^+ \rangle,$$

with

$$v(q; \omega_{\alpha J^\pi}) = \frac{2}{\pi} \frac{1}{q(q + \omega_{\alpha J^\pi})} \quad (1)$$

being the neutrino potential, and

$$\omega_{\alpha J^\pi} \cong E_{\alpha J^\pi} - (E_i + E_f)/2.$$

Here R is the nuclear radius introduced to make $\mathcal{M}_{0\nu}$ dimensionless, the symbol \otimes stands for the scalar product of spin matrices as defined in ref. [43] (with $\sigma^S = 1$ for $S = 0$, $\sigma^S = \sigma$ for $S = 1$ and $\hat{S} = \sqrt{2S+1}$), the summation goes over all virtual states $| \alpha J^\pi M \rangle$, and E_i , $E_{\alpha J^\pi}$ and E_f are the energies of the initial, intermediate and final state, respectively.

After integrating over angles and summing up over M , we get

$$\begin{aligned}\mathcal{M}_{0\nu} &= \sum_{LSJ^\pi} m_{0\nu}(L, S, J^\pi) \equiv 4\pi R \sum_{\alpha LSJ^\pi} (-)^S \int dq q^2 v(q; \omega_{\alpha J^\pi}) \\ &\times \langle 0_f^+ || \mathcal{O}_+(q\mathbf{r}_1; LSJ) || \alpha J^\pi \rangle \langle \alpha J^\pi || \mathcal{O}_+(q\mathbf{r}_2; LSJ) || 0_i^+ \rangle,\end{aligned}$$

where the quantities

$$\mathcal{O}_+(q\mathbf{r}; LSJ) = i^L j_L(qr)(Y_L \otimes \sigma^S)_J \mathbf{t}_+,$$

are one-body operators.

Within the QRPA formulation presented in ref. [10] the energies $\omega_{\alpha J^\pi}$ are solutions of the QRPA equation, and the transition matrix elements are given by

$$\begin{aligned}\langle \alpha J^\pi || \mathcal{O}_+(q\mathbf{r}; LSJ) || 0_i^+ \rangle &= - \sum_{\text{pn}} \langle \text{p} || O(q\mathbf{r}; LSJ) || \text{n} \rangle \Lambda_+(\text{pn}; \alpha J^\pi) \\ \langle 0_f^+ || \mathcal{O}_+(q\mathbf{r}; LSJ) || \alpha J^\pi \rangle &= - \sum_{\text{pn}} \langle \text{p} || O(q\mathbf{r}; LSJ) || \text{n} \rangle \Lambda_-^*(\text{pn}; \alpha J^\pi),\end{aligned}$$

where the reduced pn form factors are

$$\sqrt{4\pi} \langle \text{p} || O(q\mathbf{r}; LSJ) || \text{n} \rangle = W_{\text{pn}}^{LSJ} R_{\text{pn}}^L(q),$$

with the radial part

$$R_{\text{pn}}^L(q) = \int_0^\infty u_{\text{n}}(r) u_{\text{p}}(r) j_L(qr) r^2 dr,$$

$u(r)$ being the single-particle radial wave functions, and the angular part

$$W_{\text{pn}}^{LSJ} = i^{\ell_{\text{n}} - \ell_{\text{p}} + L} \sqrt{2} \hat{J} \hat{S} \hat{L} \hat{j}_{\text{p}} \hat{j}_{\text{n}} \hat{\ell}_{\text{n}} (\ell_{\text{n}} 0 L 0 | \ell_{\text{p}} 0) \begin{Bmatrix} \ell_{\text{n}} & \frac{1}{2} & j_{\text{n}} \\ L & S & J \\ \ell_{\text{p}} & \frac{1}{2} & j_{\text{p}} \end{Bmatrix}.$$

The amplitudes $\Lambda_{\pm}(\text{pn}; \alpha J^\pi)$ are defined as:

$$\begin{aligned}\Lambda_+(\text{pn}; \alpha J) &= \sqrt{\rho_{\text{p}} \rho_{\text{n}}} [u_{\text{p}} v_{\text{n}} X_{\text{pn}; \alpha J} + \bar{v}_{\text{p}} \bar{u}_{\text{n}} Y_{\text{pn}; \alpha J}], \\ \Lambda_-(\text{pn}; \alpha J) &= \sqrt{\rho_{\text{p}} \rho_{\text{n}}} [\bar{v}_{\text{p}} \bar{u}_{\text{n}} X_{\text{pn}; \alpha J} + u_{\text{p}} v_{\text{n}} Y_{\text{pn}; \alpha J}],\end{aligned}$$

where the unbarred (barred) quantities indicate that the quasiparticles are defined with respect to the initial (final) nucleus; $\rho_p^{-1} = u_p^2 + \bar{v}_p^2$, $\rho_n^{-1} = \bar{u}_n^2 + v_n^2$, and all the remaining notation has the standard meaning [11, 22]. As such, we obtain

$$m_{0\nu}(J^\pi) = \sum_{\alpha p n p' n'} \Lambda_+(p n; \alpha J^\pi) \Lambda_-^*(p' n'; \alpha J^\pi) \times \sum_{LS} (-)^S W_{pn}^{LSJ} W_{p'n'}^{LSJ} \mathcal{R}^L(p n p' n'; \omega_{\alpha J^\pi}), \quad (2)$$

with

$$\mathcal{R}^L(p n p' n'; \omega_{\alpha J^\pi}) = R \int_0^\infty dq q^2 v(q; \omega_{\alpha J^\pi}) R_{pn}^L(q) R_{p'n'}^L(q). \quad (3)$$

The expression (2), for the evaluation of the 0ν moments within the QRPA, is much simpler, regarding the angular momentum recoupling, than the ones currently used in the literature [3, 8, 18, 21]. The radial integrals (3) are of the same type as the ones that appear in the work of Horie and Sasaki [57], so we use their method to evaluate these, without getting involved with Moshinsky brackets. To show the simplicity of this procedure, a few results relevant to this work are reviewed in appendix A.

In the present work we consider both the GT and Fermi 2ν transition matrix elements with the corresponding amplitudes written in the form:

$$\begin{aligned} \mathcal{M}_{2\nu} &= \sum_{J=0,1} m_{2\nu}(J^+) \\ &\equiv \sum_{J=0,1} (-)^J \sum_{\alpha p n p' n'} \Lambda_+(p n; \alpha J) \Lambda_-^*(p' n'; \alpha J) W_{pn}^{0JJ} W_{p'n'}^{0JJ} / \omega_{\alpha J^+}. \end{aligned} \quad (4)$$

The corresponding total β^\mp transition strengths are:

$$\mathcal{S}_\pm = \sum_\alpha \left| \sum_{pn} \Lambda_\pm(p n; \alpha J) W_{pn}^{0JJ} \right|^2. \quad (5)$$

For the discussion of the numerical results the following unperturbed moments are also needed:

$$m_{0\nu}^0(J^\pi) = \sum_{pnLS} (-)^S \Lambda_+^0(p n) \Lambda_-^0(p n) \left(W_{pn}^{LSJ} \right)^2 \mathcal{R}^L(p n p n; \omega_{pnJ^\pi}^0), \quad (6)$$

and

$$m_{2\nu}^0(J^+) = (-)^J \sum_{\text{pn}} \Lambda_+^0(\text{pn}) \Lambda_-^0(\text{pn}) \left(W_{\text{pn}}^{0JJ} \right)^2 / \omega_{\text{pn}J^+}^0, \quad (7)$$

where $\omega_{\text{pn}J^\pi}^0$ are the unperturbed pn-energies for a given J^π , and

$$\Lambda_+^0(\text{pn}) = \sqrt{\rho_{\text{p}}\rho_{\text{n}}} u_{\text{p}} v_{\text{n}}, \quad \Lambda_-^0(\text{pn}) = \sqrt{\rho_{\text{p}}\rho_{\text{n}}} \bar{u}_{\text{n}} \bar{v}_{\text{p}}.$$

Finally, when the RHC mechanism is not considered, the half lives for the $0^+ \rightarrow 0^+$ transitions read

$$T_{1/2} = \mathcal{G}^{-1}(\mathcal{MF})^{-2},$$

where \mathcal{G} is a kinematical factor [58, 59], \mathcal{M} is the nuclear matrix element, and the values of \mathcal{F} are

$$\mathcal{F} = \begin{cases} 1 & \text{for } \beta\beta_{2\nu}, \\ \frac{\langle m_\nu \rangle}{m_e} & \text{for } \beta\beta_{0\nu}. \end{cases}$$

3 Numerical Results and Discussion

3.1 Unperturbed Matrix Elements

As in our previous studies of the $\beta\beta$ -decays [9, 10, 11, 12, 22], the numerical calculations are performed with a δ -force (in units of $MeV fm^3$)

$$V = -4\pi(v_s P_s + v_t P_t)\delta(r),$$

with different strength constants v_s and v_t for the particle-hole, particle-particle and pairing channels.

For the nuclei ^{76}Ge , ^{82}Se , ^{128}Te , ^{130}Te and ^{100}Mo , we work in an eleven dimensional model space including all the single particle orbitals of oscillator shells $3\hbar\omega$ and $4\hbar\omega$ plus the $0h_{9/2}$ and $0h_{11/2}$ orbitals from the $5\hbar\omega$ oscillator shell. The single particle energies (s.p.e.), as well as the parameters $v_s^{pair}(\text{p})$ and $v_s^{pair}(\text{n})$, have been fixed by the procedure employed in ref. [10] (i.e., by fitting the experimental pairing gaps to a Wood-Saxon potential well).

In fact, for the nuclei ^{76}Ge , ^{82}Se , ^{128}Te , and ^{130}Te we use here the same parameterization as in ref. [10].

For the nucleus ^{48}Ca the calculations have been done with three different model spaces, namely those including all the orbitals in the major shells: $2\hbar\omega$ and $3\hbar\omega$ (*space A*), $0\hbar\omega$ to $3\hbar\omega$ (*space B*), and $0\hbar\omega$ to $4\hbar\omega$ (*space C*). Experimental single particle energies have been used for the orbitals, $1p_{1/2}$, $0f_{5/2}$, $1p_{3/2}$, $0f_{7/2}$, $1s_{1/2}$ and $0d_{3/2}$, here. For the remaining orbitals a single-particle energy spacing of $\hbar\omega = 41A^{-1/3}$ MeV is assumed.

The unperturbed 2ν and 0ν moments, given by eqs. (6) and (7) and displayed in table 2, represent the upper limit to the perturbed moments. We discuss these first, in order to gain an insight into both the magnitude of $\beta\beta$ moments and the role played by the size of the configuration space. The 0ν matrix elements have been calculated by taking 5 MeV as the mean excitation energy $\omega_{\alpha J^\pi}$ in the neutrino potential (1). It has been found that the largest fraction of the 0ν strengths for all the nuclei is concentrated around this energy.

We may note that, while the $m_{2\nu}^0(J^\pi)$ matrix elements are practically unchanged as we go to larger configuration spaces, the $m_{0\nu}^0(J^\pi)$ matrix elements, and in particular those with $J \geq 2$, vary significantly. The total moment $\mathcal{M}_{0\nu}^0$, on the other hand, shows only a small variation, due to the dominance of the $m_{0\nu}^0(0^+)$ and $m_{0\nu}^0(1^+)$ in comparison with the moments $m_{0\nu}^0(J \geq 2)$. This result is not surprising as, from the study of the charge-exchange resonances, we already know that for ^{48}Ca the spaces A and B are complete spaces only for the Fermi and GT transitions. For the first-forbidden resonances ($L = 1, J^\pi = 0^-, 1^-, 2^-$), a calculation that does not include the $4\hbar\omega$ shell as well, would be totally unacceptable. In the same way for the evaluation of the second-forbidden resonances with $L = 2$ and $J^\pi = 1^+, 2^+, 3^+$ one should include the orbitals of the $5\hbar\omega$ oscillator shell (otherwise the corresponding charge exchange sum rule will not be satisfied), and so on. Fortunately, the problem with the size of the configuration space is not so serious for the $\beta\beta$ -decay. The reason is that the transition amplitude for this process is proportional to the factor $u_p v_n \bar{u}_n \bar{v}_p$ that rapidly decreases as we move away from the valence orbitals. The same argument is not valid for charge-exchange transitions whose amplitudes carry the factor $u_p v_n$ or the factor $\bar{u}_n \bar{v}_p$.

3.2 Perturbed Matrix Elements

The particle-hole channel parameter values, $v_s^{ph} = 27$ and $v_t^{ph} = 64$ for ^{48}Ca and $v_s^{ph} = 55$ and $v_t^{ph} = 92$ for the remaining six nuclei, have been taken from a study of energy systematics of the GT resonances ([39, 60]. For further discussion it is convenient to introduce the parameters s and t , defined as the ratios between the $T = 1$, $S = 0$ and $T = 0$, $S = 1$ coupling constants in the PP channels and the pair ing force constants, i.e.,

$$s = \frac{2v_s^{pp}}{v_s^{pair}(\text{p}) + v_s^{pair}(\text{n})} \quad ; \quad t = \frac{2v_t^{pp}}{v_s^{pair}(\text{p}) + v_s^{pair}(\text{n})}.$$

For a value of $s \cong 1$ the isospin symmetry is restored within the QRPA, leading to a concentration of $\mathcal{S}_+(0^+)$ strength in a single state and resulting in

$$\mathcal{S}_-(0^+) \cong 0 \ ; \ m_{2\nu}(0^+) \cong 0, \text{ and } m_{0\nu}(0^+) \cong 0. \quad (8)$$

From fig. 1 it is evident that the conditions (8) are fulfilled reasonably well for all the nuclei discussed here. As such we fix the spin-singlet PP strength at the value $s = 1$, obtaining $m_{2\nu}(0^+) \cong 0$ and $\mathcal{M}_{2\nu} \cong m_{2\nu}(1^+)$. It should be noted that in most of the QRPA calculations performed so far, the moment $m_{2\nu}(0^+)$ has been simply ignored by invoking the isospin symmetry. Simultaneously, however, sizable values for the moment $m_{0\nu}(0^+)$ have been reported. Apparently such calculations are inconsistent since the restoration of the isospin symmetry within the QRPA takes place at the level of the residual interaction and not at the level of the one body operator t_+ .

Before looking at the experimental data we consider the question of validity of the BCS approximation for far off orbitals as the configuration space is extended. With this in mind, three different calculations have been performed for ^{48}Ca , namely

Calculation I: Both the BCS and the QRPA equations have been solved within the single-particle space A.

Calculation II: The configuration space for solving QRPA equations extends over the major shells $0\hbar\omega - 4\hbar\omega$ (space C), but the gap equations have been solved within the space B only. The $4\hbar\omega$ shell is assumed to be totally empty.

Calculation III: The full space C has been used at all steps of the calculation.

In each case, the pairing interaction strengths have been determined by fitting the experimental odd-even mass differences for the relevant nuclei. The calculated $\mathcal{S}_-(1^+)$, $-m_{2\nu}(1^+)$, $-m_{0\nu}(1^+)$ and $-\mathcal{M}_{0\nu}$, as a function of parameter t , are shown in fig. 2. The curves have been drawn up to the first pole of $m_{2\nu}(1^+)$, where the energy of the lowest virtual 1^+ state becomes equal to the energy of the initial or final state and the QRPA breaks down. The magnitudes of all the quantities mentioned above depend to some extent on the size of the configuration space and the way in which the BCS equations have been solved. Yet, the general trend is the same in the three spaces. As a matter of fact, if one fixes the value of the parameter t at the lowest value of $\mathcal{S}_-(1^+)$, we get practically the same results from all three calculations; that is: $m_{2\nu}(1^+) \cong 0.09 \text{ MeV}^{-1}$, $m_{0\nu}(1^+) \cong 0.9 \text{ MeV}^{-1}$ and $\mathcal{M}_{0\nu} \cong -0.45 \text{ MeV}^{-1}$.

We may generalize the conclusions drawn for the case of ^{48}Ca . That is, an enlargement of the configuration space, beyond two major oscillator shells, is not expected to modify in essence the model predictions. As such the 11 single-particle space described earlier should be a rather complete space for the description of the $\beta\beta$ processes in the remaining five nuclei. Fig. 3 shows the relevant results for the nuclei ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te . From an inspection of figs. 2 and 3 the following common points may be made:

- 1) The minimum of $\mathcal{S}_-(1^+)$ always occurs at a value of $t = t_{sym}$ that lies between the zero and the pole of $m_{2\nu}(1^+)$.
- 2) As pointed out earlier in a similar context ([1, 2, 3, 4, 6, 8]), in the vicinity of t_{sym} , the moment $m_{0\nu}(1^+)$ is very sensitive to small variations of the parameter t , while the total moment $\mathcal{M}_{0\nu}$ is only moderately influenced by the same variation. The only exception is the nucleus ^{48}Ca . Here, due to a very pronounced dominance of the $m_{0\nu}(1^+)$ moment over the moments with higher multipolarities, the total 0ν matrix element passes through zero at a value of t that is very close to t_{sym} .

3.3 Comparison with Experimental Data

The measured 2ν and 0ν half-lives are listed in table 3 along with the values of kinematical factors $G^{2\nu}$ and $G^{0\nu}$, appropriately renormalized for an effective axial-vector coupling

constant $g_A = -g_V$, and the resulting observables $|\mathcal{M}_{2\nu}|$ and $|\mathcal{M}_{0\nu}^{\leq m_\nu}|$.

The results of the calculations for the matrix elements $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$, as well as for the predicted 2ν half-lives and the neutrino masses, are shown in table 4. From the upper panel of this table it is seen that when $t = t_{sym}$ the calculated 2ν matrix element for ^{48}Ca does not contradict the experimental limit and that the measurement of the moment $\mathcal{M}_{2\nu}$ in ^{82}Se is well accounted for by the theory. But, the calculated matrix elements $\mathcal{M}_{2\nu}$ turn out to be too small for ^{76}Ge and ^{100}Mo and too large for ^{128}Te and ^{130}Te (in both the cases by a factor of ≈ 3). It is worth noting that the measured values of $|\mathcal{M}_{2\nu}|$ for ^{76}Ge and ^{100}Mo nuclei are not very different from the calculated ones even when no PP interaction is included (see fig. 3). From the same figure, we may notice that the minima of $\mathcal{S}_-(1^+)$ are not so well defined and it is precisely near these minima that the calculated values of $\mathcal{M}_{2\nu}$ vary rather abruptly. Evidently, as such, the experimental data for the 2ν half-lives can be reproduced in all the six nuclei with a value of t very close to $t = t_{sym}$. Finally, the calculated $\mathcal{M}_{0\nu}$ moments (and estimated neutrino mass limits) for t -values that reproduce the measured matrix elements $\mathcal{M}_{2\nu}$, when these are assumed to be positive ($t = t_\uparrow$) and negative ($t = t_\downarrow$), are listed in middle and lower panels of table 4, respectively. One notices that in all the cases $t_{sym} \cong t_\uparrow$, and t_{sym} and t_\uparrow lead to practically the same upper limits for effective neutrino mass $\langle m_\nu \rangle \cong 1 \text{ eV}$.

4 Conclusions

We have calculated the 2ν and 0ν double beta observables for the nuclei ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te in the framework of the QRPA model satisfying the constraints imposed on the particle-particle coupling strengths by the isospin and SU(4) symmetries, i.e., $s = 1$ and $t \cong t_{sym}$. With the parameter t equal to t_{sym} the calculations are consistent with the experimental limit for the 2ν lifetime of ^{48}Ca and they correctly reproduce the 2ν lifetime of ^{82}Se . For the remaining nuclei the 2ν half lives are either overestimated (for ^{76}Ge and ^{100}Mo) or underestimated (for ^{128}Te and ^{130}Te) by an order of magnitude. This does not imply that one has to forgo the idea of reconstructing the SU(4) symmetry by the residual interaction. We believe that the restoration of both the isospin and SU(4) symmetries is a

genuinely useful feature of QRPA and undoubtedly plays an important role in the intricate physics involved in $\beta\beta$ processes. One also should bear in mind that: i) with a comparatively small variation ($< 10\%$) of t with respect to t_{sym} , i.e., with $t = t_{\uparrow}$ it is possible to account for the $T_{1/2}^{2\nu}$ in all the cases, and ii) the minimum value of the GT $\mathcal{S}_-(1^+)$ strength critically depends on the spin-orbit splitting over which we still do not have a complete control. In the same context it should be interesting to study the role of forbidden virtual states, particularly those with $J^\pi = 1^-$ and 2^- (see table 2), in building up the total $\mathcal{M}_{2\nu}$ moments [63].

For a long time we have been worrying about the completeness of the virtual states in the evaluation of the 0ν moments, or in other words: how the 0ν moments depend on the size of the configuration space? The results displayed in table 2 and fig. 2 seem to suggest that the enlargement of the space beyond two oscillator shells does not have a significant effect either on the GT strength $\mathcal{S}_-(1^+)$ or on the $\beta\beta$ moments $m_{2\nu}(1^+)$, $m_{0\nu}(1^+)$ and $\mathcal{M}_{0\nu}$. In the first case the configuration space turns out to be sufficiently complete while in the second case the pairing factor dependence of the $\beta\beta$ transition amplitudes inhibits the far off orbitals to contribute. We feel, however, that this question is not yet resolved unambiguously.

A final and rather general comment is in order. Besides the issue of the procedure adopted for fixing the particle-particle strength parameter, there are some additional problems within the QRPA calculations of the matrix element $\mathcal{M}_{2\nu}$, as yet not fully understood. They are related with the type of force, choice of the single particle spectra, treatment of the difference between the initial and final nuclei, etc. All these things are to some extent uncertain and therefore it is open to question whether it is possible, at the present time, to obtain a more reliable theoretical estimate for the 2ν half lives that the one reported here. Similar remarks stand for the $\mathcal{M}_{0\nu}$ moments and hence for the neutrino mass limits. The difference in a factor of about 2 – 3 between both: i) the results obtained by the Pasadena group and the groups of Tübingen and Heidelberg for ^{76}Ge and ^{82}Se nuclei, and ii) the previous and present calculations for ^{100}Mo , ^{128}Te and ^{130}Te nuclei, is just a reflection of the unavoidable uncertainty of the QRPA calculations, and it is difficult to assess which one is "better" and which is "worse".

A Radial Matrix Elements for the $0\nu\beta\beta$ -decay

Within the Horie-Sasaki formalism [57] the radial matrix elements (3) for the harmonic oscillator wave functions read as

$$\begin{aligned}\mathcal{R}^L(\text{pnp}'\text{n}'; \omega_{\alpha J\pi}) &= \text{R}[M(\text{p}, \text{n})M(\text{p}', \text{n}')]^{-\frac{1}{2}} \\ &\times \sum_{mm'} a_m(\text{p}, \text{n}) a_{m'}(\text{p}', \text{n}') f^L(m, m'; \omega_{\alpha J\pi}),\end{aligned}$$

with

$$\begin{aligned}M(n\ell, n'\ell') &= 2^{n+n'} n! n'! (2\ell + 2n + 1)!! (2\ell' + 2n' + 1)!!, \\ a_{\ell+\ell'+2s}(n\ell, n'\ell') &= \sum_{(k+k'=s)} \binom{n}{k} \binom{n'}{k'} \frac{(2\ell + 2n + 1)!! (2\ell' + 2n' + 1)!!}{(2\ell + 2k + 1)!! (2\ell' + 2k' + 1)!!}, \\ f^L(m, m'; \omega_{\alpha J\pi}) &= \sum_{\mu} a_{2\mu} \left(\frac{m-L}{2} L, \frac{m'-L}{2} L \right) \mathcal{J}_{\mu}(\omega_{\alpha J\pi}),\end{aligned}$$

and

$$\mathcal{J}_{\mu}(\omega_{\alpha J\pi}) = (2\nu)^{-\mu} \int_0^{\infty} dq q^{2\mu+2} \exp(-q^2/2\nu) v(q; \omega_{\alpha J\pi}),$$

where $\nu = M\omega/\hbar$ is the oscillator parameter. For $v(q; \omega_{\alpha J\pi})$ given by eq. (1), Tomoda et al. [64] have obtained the following recurrence relation for the momentum space integrals

$$\begin{aligned}\mathcal{J}_{\mu}(u) &= \sqrt{\frac{2\nu}{\pi}} \frac{(2\mu-1)!!}{2^{\mu}} - \frac{\sqrt{2\nu}}{\pi} u(\mu-1)! + u^2 \mathcal{J}_{\mu-1}(u), \\ \mathcal{J}_0(u) &= \sqrt{\frac{2\nu}{\pi}} - \frac{2\sqrt{2\nu}}{\pi} u \Phi(u),\end{aligned}$$

where $u = \omega_{\alpha J\pi}/\sqrt{2\nu}$ and

$$\Phi(u) = \int_0^{\infty} \frac{\exp(-t^2)}{t+u} dt = \exp(-u^2) \left[\sqrt{\pi} \int_0^{\infty} \exp(t^2) dt - \frac{1}{2} Ei(u^2) \right].$$

When finite nucleon size (FNS) effect and the short range (SR) two-nucleon correlations are included, the potential $v(q; \omega_{\alpha J\pi})$ takes the form

$$v_{FNS+SR}(q; \omega_{\alpha J\pi}) = v_{FNS}(q; \omega_{\alpha J\pi}) - \Delta v(q) + \Delta' v(q),$$

with

$$v_{FNS}(q; \omega_{\alpha J\pi}) = v(q; \omega_{\alpha J\pi}) \left(\frac{\Lambda^2}{\Lambda^2 + q^2} \right)^4, \quad \Delta v(q) = \frac{2\pi}{qq_c} \ln \left| \frac{q + q_c}{q - q_c} \right|,$$

$$\Delta v'(q) = \frac{\pi}{qq_c} \left[\sum_{n=1}^3 \frac{1}{n} (x_-^n - x_+^n) + \ln \left(\frac{x_-}{x_+} \right) \right]; \quad x_{\pm} = \frac{\Lambda^2}{\Lambda^2 + (q \pm q_c)^2},$$

where $\Lambda = 850 MeV$ is the cutoff for the dipole form factor and $q = 3.93 fm^{-1}$ is roughly the Compton wavelength of the ω -meson. The corresponding integrals $\mathcal{J}_{\mu}(\omega_{\alpha J\pi})$ have to be evaluated numerically.

References

- [1] P. Vogel and M.R. Zirnbauer, Phys. Rev. Lett. **57** (1986) 731
- [2] O. Civitarese, A. Faessler and T. Tomoda, Phys. Lett. **B194** (1987) 11
- [3] T. Tomoda and A. Faessler, Phys. Lett. **B199** (1987) 475
- [4] J. Engel, P. Vogel and M.R. Zirnbauer, Phys. Rev. **C37** (1988) 731
- [5] J. Engel, P. Vogel, O. Civitarese and M.R. Zirnbauer, Phys. Lett. **B208** (1988) 187
- [6] K. Muto and H.V. Klapdor, Phys. Lett. **B201** (1988) 420
- [7] K. Muto and H.V. Klapdor, in *Neutrinos*, ed. H.V. Klapdor (Springer-Verlag Berlin, 1988)
- [8] K. Muto, E. Bender and H.V. Klapdor, Z. Phys. **A334** (1989) 187
- [9] J. Hirsch and F. Krmpotić, Phys. Rev. **C41** (1990) 792
- [10] J. Hirsch, E. Bauer and F. Krmpotić, Nucl. Phys. **A516** (1990) 304
- [11] J. Hirsch and F. Krmpotić, Phys. Lett. **B246** (1990) 5
- [12] F. Krmpotić, in *Lectures on Hadron Physics*, ed. E. Ferreira (World Scientific, Singapore, 1990) 205
- [13] G. Pantis and J.D. Vergados, Phys. Lett. **B242** (1990) 1
- [14] J. Suhonen, S.B. Khadkikar and A. Faessler, Phys. Lett. **B237** (1990) 8
- [15] A. Staudt, T.T.S. Kuo and H.V. Klapdor-Kleingrothaus, Phys. Lett. **B242** (1990) 17
- [16] A. Staudt, K. Muto and H.V. Klapdor-Kleingrothaus, Europhys. Lett. **13** (1990) 31
- [17] O. Civitarese, A. Faessler, J. Suhonen and X.R. Wu, Nucl. Phys. **A524** (1991) 404; Phys. Lett. **B251** (1991) 333

- [18] K. Muto, E. Bender and H.V. Klapdor-Kleingrothaus, Z. Phys. **A339** (1991) 435
- [19] J. Suhonen, S.B. Khadkikar and A. Faessler, Nucl. Phys. **A529** (1991) 727
- [20] J. Suhonen, S.B. Khadkikar and A. Faessler, Nucl. Phys. **A535** (1991) 509
- [21] T. Tomoda, Rep. Prog. Phys. **54** (1991) 53
- [22] F. Krmpotić, J. Hirsch and H. Dias, Nucl. Phys. **A542** (1992) 85
- [23] A. Balysh, M. Beck, S.T. Belyaev, J. Bockholt, J. Echter Nach, A. Gurov, G. Heusser, M. Hirsch, H.V. Klapdor-Kleingrothaus, I. Kondratenko, V.I. Lebedev, B. Maier, A. Müller, F. Petry, A. Piepke, U. Schmidt-Rohr, H. Strecker, and K. Zuber, Phys. Lett. **B283** (1992) 32
- [24] K. You, Y. Zhu, J. Lu, H. Sun, W. Tian, W. Zhao, Z. Zheng, M. Ye, C. Ching, T. Ho, F. Cui, C. Yu and G. Jiang, Phys. Lett. **B265** (1991) 53
- [25] S.R. Elliot, A.A. Hahn, M.K. Moe, M.A. Nelson and M.A. Vient Phys. Rev. **46** (1992) 1535
- [26] H. Ejiri, K. Fushimi, T. Kamada, H. Kinoshita, H. Kobiki, H. Ohsumi, K. Okada, H. Sano, T. Shibata, T. Shima, N. Tanabe, J. Tanaka, T. Taniguchi, T. Watanabe and N. Yamamoto Phys. Lett. **B258** (1991) 17
- [27] T. Bernatowitz, J. Brannon, R. Brazlle, R. Cowsik, C. Hohenberg, and F. Podosek, Phys. Rev. Lett. **69** (1992) 2341; Phys. Rev. **C47** (1993) 806
- [28] A. Alessandrello, C. Broffero, D.V. Camin, O. Cremonesi, E. Fiorini, G. Gervasio, A. Guilian, M. Pavan, G. Pessina, E. Previtali and L. Zanolli, Phys. Lett. **B285** (1992) 176
- [29] A. Griffiths and P. Vogel, Phys. Rev. **C46** (1992) 181

- [30] H. Seyfarth, H.H. Güven, B. Kardon, W.D. Lauppe, G. Lhersonneau, K. Sistemich, S. Brant, N. Kaffrell, P. Maier-Komor, H.K. Vonach, V. Paar, D. Vorkapić and R.A. Meyer, Z. Phys. **A339** (1991) 269
- [31] D.J. Thouless, Nucl. Phys. **22** (1960) 78
- [32] G.E. Brown, *Unified Theory of Nuclear Models* (North-Holland, Amsterdam, 1964) p. 53
- [33] C.A. Engelbrecht and R.H. Lemmer, Phys. Rev. Lett. **24** (1970) 607
- [34] H.C. Lee, Phys. Rev. Lett. **27** (1971) 200
- [35] A.M. Lane and J. Martorell, Ann. Phys. **129** (1980) 273
- [36] D.E. Bauinum, J. Rapaport, C.D. Goodman, D.J. Horen, C.C. Foster, M.D. Greenfield, C.A. Goulding, Phys. Rev. Lett. **44** (1980) 1751
- [37] D.J. Horen, C.D. Goodman, C.C. Foster, C.A. Goulding, M.D. Greenfield, J. Rapaport, D.E. Bauinum, E. Sugarbaker, T.G. Masterson, F. Petrovich and W.G. Love, Phys. Lett. **B95** (1980) 27
- [38] D.J. Horen, C.D. Goodman, D.E. Bainum, C.C. Foster, C. Gaarde, C.A. Goulding, M.D. Greenfield, J. Rapaport, T.N. Taddeucci, E. Sugarbaker, T.G. Masterson, S.M. Austin, A. Galonsky and W. Sterrenburg, Phys. Lett. **B99** (1981) 383
- [39] K. Nakayama, A. Pio Galeão and F. Krmpotić, Phys. Lett. **B114** (1982) 217
- [40] Yu.V. Gaparov, Yu.S. Lyutostanskii and V.G. Aleksankin, Pis'ma Zh. Exsp. Teor. Fiz. **34** (1981) 407
- [41] T. Suzuki, Nucl. Phys. **A379** (1982) 110
- [42] J.I. Fujita and K. Ikeda, Nucl. Phys. **67** (1965) 143

- [43] A. Bohr and B.R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969) vol. 1, p. 348.
- [44] J. Bernabeu, B. Desplanques, J. Navarro and S. Noguera, *Z. Phys.* **C46** (1990) 323
- [45] Yu.V. Gaparov, N.M. Shulgina and D.M. Vladimirov, *Nuc. Phys.* **A391** (1982) 93
- [46] P. Vogel, M. Ericson and J.D. Vergados, *Phys. Lett.* **B212** (1988) 259
- [47] M. Hino, K. Muto and T. Oda, *Phys. Rev.* **C37** (1988) 1328
- [48] G. Mairle, K.T. Knopfle and M. Seeger, *Nucl. Phys.* **A490** (1988) 371
- [49] D.C. Zheng, L. Zamick and N. Auerbach, *Phys. Rev.* **C40** (1989) 936; *Ann. Phys.* **197** (1990) 343
- [50] K. Muto, *Phys. Lett.* **B277** (1992) 13
- [51] P. Vogel and W.E. Ormand, *Phys. Rev.* **C47** (1993) 623
- [52] G.B. Gelmini and M. Roncadelli, *Phys. Lett.* **B99** (1981) 411
- [53] H. Georgi, S.L. Glashow and S. Nussinov, *Nuc. Phys.* **B193** (1981) 297
- [54] R.N. Mohapatra and E. Tagasugi, *Phys. Lett.* **211B** (1988) 192
- [55] G.S. Abrams et al., *Phys. Rev. Lett.* **63** (1989) 724
- [56] M.C. Gonzalez-Garcia and Y. Nir, *Phys. Lett.* **B232** (1989) 383
- [57] H. Horie and K. Sasaki, *Prog. Theor. Phys.* **25** (1961) 475
- [58] M. Doi, T. Kotani and E. Takasugi, *Prog. Theor. Phys. Suppl.* **83** (1985) 1
- [59] M. Doi, T. Kotani and E. Takasugi, *Phys. Rev.* **D37** (1988) 2104
- [60] H. Castillo and F. Krmpotić, *Nucl. Phys.* **A469** (1987) 637

- [61] R.K. Bardin, P.J. Gollon, J.D. Ullman and C.S. Wu, Nuc. Phys. **A 158** (1970) 337
- [62] F.T. Avignone III, R.L. Brodzinsky, C.K. Guerard, I.V. Kirpichnikov, H.S. Miley, V.S. Pogosov, J.H. Reeves, A.S. Starostin and A.G. Tamanyan, Phys. Lett. **B256** (1991) 559
- [63] A. Williams and W.C. Haxton, in *Intersections between Particle and Nuclear Physics*, ed. G.M. Bunce (AIP Conf. Proc. No. 176, 1988) p. 924.
- [64] T. Tomoda, A. Faessler, K.W. Schmid and F. Grümmer, Nucl. Phys. **A452**, (1986) 591

Figure Captions

Figure 1: Fermi observables $\mathcal{S}_-(J^\pi = 0^+)$, $m_{2\nu}(J^\pi = 0^+)$ (in units of $[MeV]^{-1}$) and $m_{0\nu}(J^\pi = 0^+)$ for the nuclei ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te , as a function of particle-particle $S = 0$, $T = 1$ coupling constant s .

Figure 2: Gamow-Teller observables $\mathcal{S}_-(J^\pi = 1^+)$, $m_{2\nu}(J^\pi = 1^+)$ (in units of $[MeV]^{-1}$), $m_{0\nu}(J^\pi = 1^+)$ and the total $\mathcal{M}_{0\nu}$ moment for the nucleus ^{48}Ca , as a function of the particle-particle $S = 1$, $T = 0$ coupling constant t , within the single-particle spaces A, B and C.

Figure 3: Gamow-Teller observables $\mathcal{S}_-(J^\pi = 1^+)$, $m_{2\nu}(J^\pi = 1^+)$ (in units of $[MeV]^{-1}$), $m_{0\nu}(J^\pi = 1^+)$ and the total $\mathcal{M}_{0\nu}$ moment for the nuclei ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te , as a function of the particle-particle $S = 1$, $T = 0$ coupling constant t .

Tables

Table 1: Upper bounds on the effective neutrino mass $\langle m_\nu \rangle$ (in eV) obtained from the QRPA calculations of the nuclear matrix elements. For the sake of comparison, in all the cases the effective axial vector coupling constant $g_A^{eff} = -g_V$ has been employed. This means that the results for $\langle m_\nu \rangle$ from refs. [3, 8, 18, 20] have been properly renormalized by the factor $(1.25)^2$. The experimental data for the half-lives which have been used are indicated in the second row. The two values of the Pasadena group correspond to their PP strengths: (a) $\alpha'_1 = -390$ and (b) $\alpha'_1 = -432$, in units of MeV fm^3 .

	^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{130}Te
Exp.	ref. [24]	ref. [23]	ref. [25]	ref. [26]	ref. [27]	ref. [28]
ref. [3]		2.3	8.2		2.4	24
ref. [4] (a)		4.4	20	20	1.8	22
ref. [4] (b)		10	41		2.4	29
ref. [8]		2.0	7.4	26	1.5	21
ref. [18]	22					
ref. [20]		3.1	12		3.8	31

Table 2: Unperturbed $m_{2\nu}(J^\pi)$ and $m_{0\nu}(J^\pi)$ moments in units of $[MeV]^{-1}$. As explained in the text, three different single-particle spaces have been used for the nucleus ^{48}Ca .

	^{48}Ca			^{76}Ge	^{82}Se	^{90}Mo	^{128}Te	^{130}Te
	A	B	C					
$-m_{2\nu}^0(J^\pi)$								
0^+	0.249	0.255	0.253	0.423	0.491	0.256	0.678	0.615
1^+	0.406	0.417	0.414	0.923	0.966	1.513	1.602	1.434
$-m_{0\nu}^0(J^\pi)$								
0^+	0.928	0.953	0.990	2.258	2.424	1.857	2.671	2.472
1^+	2.072	2.158	2.268	6.168	6.259	7.204	7.310	6.751
2^+	0.316	0.384	0.426	1.285	1.283	1.470	1.483	1.367
3^+	0.321	0.394	0.438	1.112	1.179	1.421	1.260	1.164
4^+	0.109	0.131	0.151	0.507	0.525	0.708	0.636	0.587
5^+	0.134	0.154	0.170	0.407	0.459	0.636	0.497	0.464
6^+	0.037	0.037	0.044	0.196	0.214	0.356	0.253	0.236
7^+	0.075	0.075	0.079	0.145	0.187	0.275	0.212	0.200
8^+	0.000	0.000	0.001	0.055	0.070	0.160	0.096	0.092
9^+	0.000	0.000	0.000	0.056	0.083	0.065	0.087	0.084
10^+	0.000	0.000	0.000	0.002	0.002	0.006	0.029	0.028
0^-	0.027	0.040	0.071	0.177	0.194	0.337	0.130	0.126
1^-	0.307	0.378	0.564	1.604	1.749	2.655	1.367	1.315
2^-	0.209	0.250	0.337	1.161	1.266	1.628	1.128	1.078
3^-	0.158	0.183	0.244	0.882	0.954	1.124	0.929	0.881
4^-	0.061	0.061	0.093	0.517	0.559	0.735	0.559	0.531
5^-	0.062	0.063	0.087	0.417	0.453	0.456	0.479	0.453
6^-	0.000	0.000	0.010	0.157	0.175	0.276	0.249	0.237
7^-	0.000	0.000	0.009	0.181	0.197	0.171	0.258	0.245
8^-	0.000	0.000	0.000	0.014	0.019	0.051	0.084	0.082
9^-	0.000	0.000	0.000	0.015	0.018	0.048	0.128	0.122
$-\mathcal{M}_{0\nu}^0$	4.816	5.261	5.983	17.317	18.268	21.641	19.844	18.516

Table 3: Values of the measured half-lives $T_{1/2}$, kinematical factors G and the experimental \mathcal{MF} -quantities for the 2ν and 0ν $\beta\beta$ decays. The G factors are those from ref. [59], but renormalized for $g_A = -g_V$.

Observable	^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{130}Te
$T_{1/2}^{2\nu}[\text{yr } 10^{20}]$	$> 0.36^a$	$9.2_{-0.4}^{+0.7^b}$	$1.08_{-0.06}^{+0.26^c}$	$0.115_{-0.020}^{+0.030^d}$	$(7.7 \pm 0.4) 10^4^e$	27 ± 1^e
$G^{2\nu}[\text{yr}(\text{MeV})^2]^{-1}$	$0.423 10^{-17}$	$0.139 10^{-19}$	$0.464 10^{-18}$	$0.101 10^{-17}$	$0.911 10^{-22}$	$0.512 10^{-18}$
$ \mathcal{M}_{2\nu} [\text{MeV}]^{-1}$	< 0.081	$0.280_{-0.010}^{+0.006}$	$0.141_{-0.014}^{+0.004}$	$0.294_{-0.033}^{+0.029}$	$0.038_{-0.01}^{+0.01}$	$0.027_{-0.01}^{+0.01}$
$T_{1/2}^{0\nu}[\text{yr } 10^{21}]$	$> 9.5^f$	$> 1400^g$	$> 27^c$	$> 4.7^d$	$> 7700^e$	$> 2.5^h$
$G^{0\nu}[\text{yr}]^{-1}$	$0.260 10^{-13}$	$0.261 10^{-14}$	$0.114 10^{-13}$	$0.187 10^{-13}$	$0.746 10^{-15}$	$0.181 10^{-13}$
$ \mathcal{M}_{0\nu}^{\frac{\leq m_\nu}{m_e}} 10^4$	< 0.64	< 0.17	< 0.57	< 1.07	< 0.13	< 1.5

^{a)} (laboratory data) ref. [61]

^{b)} (laboratory data) ref. [62]

^{c)} (laboratory data) ref. [25]

^{d)} (laboratory data) ref. [26]

^{e)} (geochemical data) ref. [27]

^{f)} (laboratory data) ref. [24]

^{g)} (laboratory data) ref. [23]

^{h)} (laboratory data) ref. [28]

Table 4: Calculated 2ν and 0ν moments and the corresponding upper limits for the effective neutrino mass $\langle m_\nu \rangle$. Here $s = 1$ and three different sets of parameter t have been considered, namely, $t = t_{sym}$, $t = t_\uparrow$ and $t = t_\downarrow$. The last two reproduce the measured 2ν matrix element (without taking error bars into consideration) when these are assumed to be positive and negative, respectively. For $t = t_{sym}$, the calculated $T_{1/2}^{2\nu}$ values are also shown.

	^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{130}Te
t_{sym}	1.50	1.25	1.30	1.50	1.40	1.40
$(\mathcal{M}_{2\nu})_{sym} [MeV]^{-1}$	0.091	0.100	0.121	0.102	0.118	0.096
$(\mathcal{M}_{0\nu})_{sym}$	-0.46	-5.7	-5.6	-6.2	-7.0	-6.6
$\langle m_\nu \rangle_{sym} [eV]$	71	1.5	5.3	8.8	1.0	12
$(T_{1/2}^{2\nu})_{sym} [yr \ 10^{20}]$	0.28	71	1.5	0.95	7.9×10^3	2.1
t_\uparrow	1.48	1.35	1.39	1.53	1.35	1.30
$(\mathcal{M}_{0\nu})_\uparrow$	-0.48	-4.5	-5.3	-6.9	-7.3	-7.0
$\langle m_\nu \rangle_\uparrow [eV]$	68	1.9	7.1	10	1.0	11
t_\downarrow	1.20	0.30	1.00	1.30	1.22	1.20
$(\mathcal{M}_{0\nu})_\downarrow$	-1.41	-10.2	-7.7	-8.3	-8.1	-7.5
$\langle m_\nu \rangle_\downarrow [eV]$	23	0.87	4.2	6.6	0.86	10

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